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A MODEL OF THE ENERGY FLUXES IN A SOLAR HEATED SWIMMING POOL AND ITS EXPERIMENTAL VALIDATION

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Abstract—In the present work, a theoretical model is presented concerning the various energy fluxes in a solar heated outdoor swimming pool, with a bare or covered surface. The analysis considers the instantaneous energy balance using actual climatological data, special attention being focussed on setting up equations for the calculation of the turbulent heat-mass transfer energy fluxes. Thus, the net heat loss can be calculated under constant water temperature conditions, paving the way for the determination of the needed solar collector heat power and/or conventional heating. A computer program is written and developed to implement the analysis. To check the validity of the model, experimental work has been carried out at the Athens Olympic (25 × 50 m) swimming pool, which is heated by a solar collector array consisting of flat plate plastic panels. A vast monitoring campaign is undertaken, involving the installation of the sensing instruments and the data acquisition and reduction system. All the relevant parameters were continuously recorded and conveniently averaged over 1 h intervals. For the first year of operation, the agreement between experimental results and theoretical calculations (on an hourly basis) is promising for the days tested. From a comparison of the magnitudes of the individual flux losses, it is concluded that the higher loss contributor is the evaporative one.

Swimming pool   Solar heating   Energy fluxes

NOMENCLATURE

A = Pool surface area (m^2)
b = Exponent in Rayleigh number
C_a = Water-vapour mass concentration in ambient air (kg/m^3)
C_p = Water-vapour mass concentration at pool surface (kg/m^3)
c = Specific heat capacity (J/kg °C)
D = Diffusivity (m^2/s)
dT = Differential temperature (K)
dt = Differential time (s)
f_x = Local friction factor
G = Energy gain from sun (J)
g = Gravity acceleration ($= 9.81\ m/s^2$)
$\dot{g}$ = Energy gain flux (specific) from sun (W/m^2)
H_{fg} = Water latent heat of evaporation (J/kg)
h = Heat transfer coefficient (W/m^2 °C)
I_b = Beam solar radiation on horizontal plane (W/m^2)
I_d = Diffused solar radiation on horizontal plane (W/m^2)
k = Thermal conductivity (W/m °C)
L = Mean pool dimension (m)
M = Pool water mass (kg)
$\dot{m}_e$ = Evaporated water mass flow rate (kg/s)
$\dot{m}_s$ = Water mass flow rate loss due to splashing and evaporation (kg/s)
P_a = Water-vapour partial pressure in ambient air (N/m^2)
P_p = Water saturation pressure at pool temperature (N/m^2)
Q = Energy loss (J)
$\dot{q}$ = Energy loss flux (specific) (W/m^2)
T = Temperature (K)
T_0 = Water supply temperature (K)
T_{sk} = Sky temperature (K)
t = Time (s)
u = Velocity (m/s)
V = Wind speed (free stream) (m/s)
x = Length or characteristic dimension (m)

Greek letters

α = Water absorption coefficient for beam radiation
$\bar{\alpha}_M$ = Mean (spatially) mass exchange coefficient (m/s)
β = Thermal expansion coefficient (1/K)
γ = Reduction factor for evaporative heat loss due to wind breaking effect
ΔT = Temperature difference (K)
ϵ = Water emissivity
ζ = Water absorption coefficient for diffused radiation
ν = Kinematic viscosity (m^2/s)
ρ = Density (kg/m^3)
σ = Stefan-Boltzmann (radiation) constant ($W/m^2\,K^4$)
τ = Time interval (step) (s)
ϕ = Relative humidity

Subscripts

a = Ambient air
c = Convection
cu = Cover upper surface
e = Evaporation
fc = Forced convection
nc = Natural convection
p = Pool water
r = Radiation
s = Splash
w = Wall (conduction)
x = Local, at distance x

Superscript

— Denotes mean value (spatially)

Dimensionless numbers

$\mathrm{Gr} = \beta g \, \Delta T x^3/\nu^2$, Grashof number
$\mathrm{Nu} = h \, x/k$, Nusselt number
$\mathrm{Pr} = \nu \, \rho \, c/k$, Prandtl number
$\mathrm{Ra} = \mathrm{Gr} \, \mathrm{Pr}$, Rayleigh number
$\mathrm{Re} = u \, x/\nu$, Reynolds number
$\mathrm{Sc} = \nu/D$, Schmidt number
$\mathrm{St} = \mathrm{Nu}/\mathrm{Re} \, \mathrm{Pr}$, Stanton number

INTRODUCTION

The natural energy (solar) input into an outdoor swimming pool, all over Greece, is insufficient for the pool to be used all the seasons of the year. For increasing pool water temperatures, either conventional heating can be used (for example oil or gas fired boilers), or heating by solar collectors [1], with the possible use of pool blankets. These last methods have proved most effective, as revealed by a computer simulation of Szeicz and McMonagle [2], especially if seen in the context of rising fuel prices or their shortage in the future.

For determining the necessary heating power for such a swimming pool, whether "conventional" or "renewable", an accurate analysis is needed for the calculation of the net heat loss. Czarnecki [3], very early, gave an expression for the total rate of heat loss per day from an open pool. Govaer and Zarmi [4] extended the analysis to include indoor pools. Recently, Govind and Sodha [5] reported on a thermal analytical model with periodic insolation and atmospheric conditions, showing substantial improvement by the placement of transparent PVC covers.

The present analysis considers the energy balance for the water body of a swimming pool, using actual hourly climatological data (solar radiation, relative humidity, air temperature, wind velocity, etc), attention being paid to modelling each energy flux, and especially the turbulent heat-mass transfer exchange fluxes, starting from first principles.

The theoretical predictions are compared with the results from laborious experimental work, involving the installation of sensing instruments and the data acquisition and reduction system, which was carried out at the Athens swimming pool, which is one of the three Olympic outdoor swimming pools heated by solar collectors in Greece, the other ones located in Macedonia and Crete [6]. This formed a part of a vast demonstration programme partly subsidized by the European Community [7]. A similar smaller programme, undertaken by the Victorian Solar Energy Council [8] for heating an indoor swimming pool, demonstrated the installation and operation of such a solar heating system supplemented by heating provided by hot water from an oil-fired boiler.

The present research group gave a brief outline of the most important features [9] of the present analysis, and also reported [10] on the preliminary results (first 2 months) obtained by operating the Athens swimming pool. The present work gives a detailed exposition of the theoretical analysis and establishes its validity with experimental results obtained from the same swimming pool, throughout its first year of operation.

THE THEORETICAL ANALYSIS

For an outdoor swimming pool of surface area A, the energy balance [1, 2] equation between the pool closed water body and the environment is written in differential form,

$$M \, c_p \, \mathrm{d}T = (\dot{g} - \dot{q}_r - \dot{q}_w - \dot{q}_c - \dot{q}_e) \, A \, \mathrm{d}t. \tag{1}$$

To keep a constant pool water temperature of 26°C ($\mathrm{d}T = 0$), which is also assumed uniform due to the filtering system, the terms on the RHS of equation (1) provide the heating power which must be supplied by the solar heating system or/and the auxiliary one. The expression for each energy flux, on the RHS of equation (1), is analyzed in the following subsections.

Bare pool

Gain flux from the sun $\dot{g}$. For the present experiments, the total solar radiation on the tilted collector (45°) plane facing south is available. Using the methods of Liu and Jordan and of Orgill and Hollands [1] in reversed order (requiring an iterative procedure), the beam I_b and diffused I_d components on the horizontal pool surface are calculated. Therefore, the energy gain from the sun, in a time interval τ, is

$$G = \int_0^\tau \dot{g} \, A \, \mathrm{d}t = \int_0^\tau (I_b \, \alpha + I_d \, \zeta) \, A \, \mathrm{d}t \tag{2}$$

where α and ζ are the water absorption coefficients for beam and diffused radiation respectively. For this calculation, the actual horizon, resulting from the various landscaping features, such as buildings and fences, was taken into account [2].

Radiative loss flux $\dot{q}_r$. The radiative (longwave) energy loss, in a time interval τ, is calculated [1] from,

$$Q_r = \int_0^\tau \dot{q}_r \, A \, \mathrm{d}t = \int_0^\tau \epsilon \, \sigma \, (T_{sk}^4 - T_p^4) \, A \, \mathrm{d}t \tag{3}$$

where the sky temperature $T_{sk} = 0.0552 \, T_a^{1.5}$, $\sigma = 5.67 \times 10^{-8}$ W/m² K⁴ and ϵ ($= 0.97$) is the water emissivity [2]. For this calculation, the landscaping features (buildings, fences etc.) were taken into account, too.

Conduction (wall) loss flux $\dot{q}_w$. Although conduction loss can be calculated by classical heat transfer methods [11, 4], it is generally found [1, 2] that its magnitude is relatively small in well built pools, which is also the case for the present work.

Convection loss flux $\dot{q}_c$. For this calculation, the Von Karman [12, 13] universal velocity profile is assumed to exist in the turbulent stream of atmospheric air, flowing parallel to the water surface, for the calculation of the forced convective heat transfer coefficient.

The turbulent boundary layer is assumed to start from the edge of the pool, since the preceding laminar layer is very small relative to the present pool dimensions, 50 × 20 m (a rough calculation, assuming a critical Reynolds number of 5×10^5 and $V = 2$ m/s, gives a laminar layer length of 3.75 m). The theory also requires $0.5 < \mathrm{Pr} < 10$, which is valid for the fluids in question.

Therefore [12], the local forced convective heat transfer coefficient h_{xcf} or its corresponding Stanton number is given by,

$$\mathrm{St}_{xfc} = \frac{0.0295\,\mathrm{Re}_x^{-0.2}}{1 + 0.172\,\mathrm{Re}_x^{-0.1}\,[5\,\mathrm{Pr} + 5\ln(5\mathrm{Pr} + 1) - 14]} \quad (4)$$

assuming an "analogy" between heat and momentum transfer and where the following substitution has been used, for the local friction factor,

$$f_x = 0.059\,\mathrm{Re}_x^{-0.2} \quad (5)$$

the length x being measured from the pool's edge.

Since expression (4) is difficult to integrate, it is approximated [13] by the following simpler expression,

$$\mathrm{St}_x\,\mathrm{Pr}^{0.4} = 0.0295\,\mathrm{Re}_x^{-0.2}. \quad (6)$$

Substituting in equation (6) the expressions for St_x and Pr, and integrating (spatially) with respect to length x, we obtain the value of the mean forced convective heat transfer coefficient,

$$\bar{h}_{xfc} = 0.0369\,\rho_a\,c_a\,\mathrm{Pr}^{-0.4}\left(\frac{L}{\nu_a}\right)^{-0.2} V^{0.8}. \quad (7)$$

For air properties at a mean temperature of 15°C and a mean pool length of 40 m (since the wind velocity has not a constant direction), equation (7) becomes,

$$\bar{h}_{fc} = 2.65\,\gamma\,V^{0.8} \quad (8)$$

in S.I. units.

In equation (8), a correction (reduction) factor γ is introduced to take into account the "windbreaking" effect of the surrounding buildings, fences, etc. [14]. Careful measurements, on the spot, with portable anemometers gave a mean reduction factor $\gamma \simeq 0.5$ with respect to the measured (unimpeded) free stream wind speed value V.

Although the natural convection heat transfer coefficient is relatively small, it is taken into account (especially when $V = 0 \rightarrow \bar{h}_{fc} = 0$) using standard correlations [15] of the form,

$$\bar{h}_{nc} = \sim(\mathrm{Gr\,Pr})^b = \sim \mathrm{Ra}^b \quad (9)$$

for a plate cooled from above ("cold" periods), or heated from above ("hot" periods), with $L = 40$ m and the air properties evaluated at a mean temperature, as before.

The total energy convection loss, in a time interval τ, is then,

$$Q_c = \int_0^\tau \dot{q}_c\,A\,\mathrm{d}t = \int_0^\tau (2.65\,\gamma\,V^{0.8} + \bar{h}_{nc})\,(T_p - T_a)\,A\,\mathrm{d}t. \quad (10)$$

Evaporation loss flux $\dot{q}_e$. For this calculation, the "analogy" is extended [12, 13] between heat and mass transfer (water exchange between pool and air). Introducing the Schmidt number Sc in place of Pr and using again $L = 40$ m and air properties at a mean temperature of 15°C, we obtain the following expression for the mean (spatially) mass exchange coefficient $\bar{\alpha}_M$ (in S.I. units),

$$\bar{\alpha}_M = 0.0306\,\gamma\,V^{0.8} \quad (11)$$

where γ has the same reasoning as in the previous section.

Therefore, the evaporated water mass rate from the pool surface is

$$\dot{m}_e = \bar{\alpha}_M\,(C_p - C_a)\,A. \quad (12)$$

Using the perfect gas state equation for air and steam and introducing the water latent heat of evaporation at 25°C ($H_{fg} = 2.4 \times 10^6$ J/kg) in equation (12), we finally find the total energy loss due to water evaporation, in a time interval τ, as follows (S.I. units),

$$Q_e = \int_0^\tau \dot{q}_e\,A\,\mathrm{d}t = \int_0^\tau \dot{m}_e\,H_{fg}\,\mathrm{d}t = \int_0^\tau 159\,\gamma\,V^{0.8}\left(\frac{P_p}{T_p} - \frac{P_a}{T_a}\right) A\,\mathrm{d}t. \quad (13)$$

For $280\mathrm{K} < T < 310\mathrm{K}$, the partial pressures, in equation (13), are calculated from the following formulae [9], convenient for computer handling,

$$P_p = 6895\,\{\exp[0.0529\,(T_p - 273) - 2] - 0.05\} \quad (14)$$

$$P_a = \phi\,6895\,\{\exp[0.0529\,(T_a - 273) - 2] - 0.05\} \quad (15)$$

where ϕ is the relative humidity expressed in fractional form.

The energy loss due to water loss from evaporation, or swimmers splashing, is also taken into account (although small in magnitude) from the equation

$$Q_s = \int_0^\tau \dot{m}_s\,c_p\,(T_p - T_0)\,\mathrm{d}t \quad (16)$$

where $\dot{m}_s$ is the replaced water mass rate from the city mains with inlet temperature T_0.

Covered pool

Although the analysis above refers to a bare pool, it is easily modified for a pool covered with a transparent floating cover (at night), noting that: (a) $\dot{q}_e = 0$, (b) $\dot{q}_w = 0$ as before, (c) $\dot{g} = 0$ (the cover is set only at night), (d) $\dot{q}_c$ is calculated from the same equation as before, where T_p is replaced by the cover's upper surface temperature T_{cu}, which is determined by an iterative solution of the three heat transfer equations (one for conduction in the cover and the other two for convection on either side of it) and (e) $\dot{q}_r$ is calculated from the same equation as before, where T_p is replaced by T_{cu}, and the final value is multiplied by the cover's transmissivity, which has a value of $\approx 50\%$ for the present application.

THE COMPUTER PROGRAM

In order to implement the theoretical analysis, a computer program was written and developed in FORTRAN IV language and run on a CDC 7000 series computer. The equations of the previous section are solved in time, with a convenient time step of $\tau = 1$ h, since all the relevant measured parameters (incident radiation, temperatures, relative humidity, etc.), which form "inputs" to the program, are averaged by the instruments over 1 h intervals.

The solution procedure is rather straightforward with the exception of two iterative procedures for calculating the gain flux from the sun and solving the heat transfer equations around the transparent pool cover, as mentioned above.

THE EXPERIMENTAL INVESTIGATION

The validity of the theoretical analysis was checked against the experimental measurements, taken during a vast experimental work carried out at the Athens swimming pool, shown in Fig. 1, which is one of the three Olympic outdoor pools in Greece heated by solar collectors [6], following precise guidelines [16] for extensive monitoring of the system.

The swimming pool heating system

The Athens swimming pool dimensions are 25 × 50 m with a depth ranging from 1.80 to 2.20 m. Figure 2 gives a schematic diagram of the pool circuit and the solar and auxiliary heating system, where the main flow and temperature measuring stations are shown. A filtering system is provided.

The solar collector array (containing air separators and vents in its circuit) consists of 217 flat plate unglazed "Fafco" polyolefin solar panels [1] connected in parallel, with a combined area of 780 m^2 and a tilt angle of 45° facing south. The heating systems are set to maintain the pool temperature at 25–27°C. The pool water is circulated directly through the solar collector array, without the intervention of a heat exchanger. A differential temperature switch energizes the installed solar pumps whenever the collector outlet header temperature exceeds the pool temperature by 2°C. The auxiliary heating system provides hot water, from an oil-fired boiler, through a heat exchanger unit. It is set into operation when the solar system is unable to maintain the pool temperature above the lower limit. Should the pool temperature rise above 27°C, a high temperature sensor stops the solar pumps, and/or the auxiliary system. It is possible to circulate the hot water of the solar collectors through a tank for

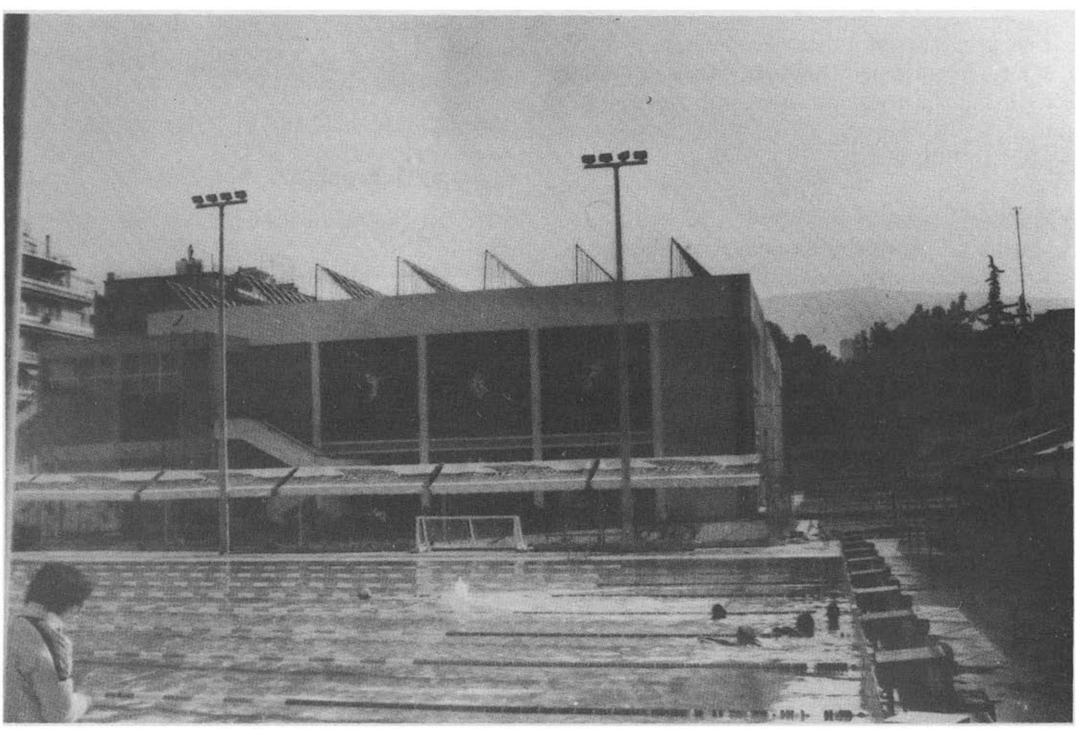

Fig. 1. Photograph of Athens Olympic swimming pool.

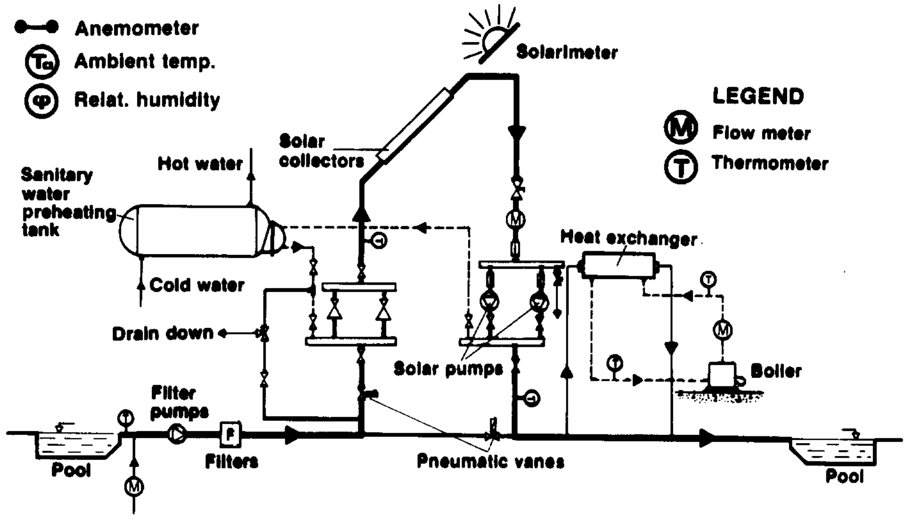

Fig. 2. Schematic diagram of Athens swimming pool system.

preheating sanitary water. During the nights, the pool surface is covered with a polyethelene plastic blanket, incorporating air bubbles, which floats on the water surface.

The monitoring instruments and procedure

A monitoring campaign was undertaken, involving the installation of the sensing instruments, the data acquisition and reduction system [6] and the processing of the experimental results, after paying a considerable amount of time for procedures which ensured early identification of faults and failures and their prompt remedy.

The measuring instruments included (see Fig. 2) temperature sensors at various locations in the circuit, flow meters, and energy (heat) meters. The temperature sensors were resistance thermometers Pt 100. Two flow meters "$E + H$ Magpac", working on the magnetic inductive principle, measure the water flow rates to the solar collectors and the boiler. They are relatively expensive, but they were preferred due to their good accuracy. The electric signals from the thermometers and flow meters enter "Flowtec" energy meters, which calculate the heat content by multiplication and integration.

The solar radiation is measured by a "Thies-Clima" pyranometer, mounted at the slope of the collectors. The ambient relative humidity is measured by a "Thies-Clima" ("K" element) hygrometer. The wind speed is measured by a "Thies-Clima" anemometer, which is placed at 9.3 m above the ground in a place far away from obstructions.

All the signals, from the above mentioned sensors, are fed into a local microprocessor unit which outputs, via a printer, all the values integrated over 1 h intervals. Also, for double checking the experimental results, all the signals from the measuring instruments are recorded on a multi-point strip chart recorder and a thermal printer. These records are very useful since they provide a measure of the variation of the measured parameters inside the 1 h interval.

The output data from the microprocessor unit and the chart recorder and thermal printer were collected daily and, after careful examination for locating possible errors, were stored in magnetic tapes at the Athens University big CDC computer. They were eventually processed, according to the theoretical analysis, by scientific personnel.

COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL RESULTS

Figure 3 shows the experimental and theoretical cumulative heat loss values, at the end of every hour, for 1 day. This is typical of all the days tested (see following Fig. 6). It was usually observed that a deviation occurred between the theoretical and experimental results during the day, which was getting very small at the end of the 24 h period. Probably, this is due to the high inertia of the system, for a

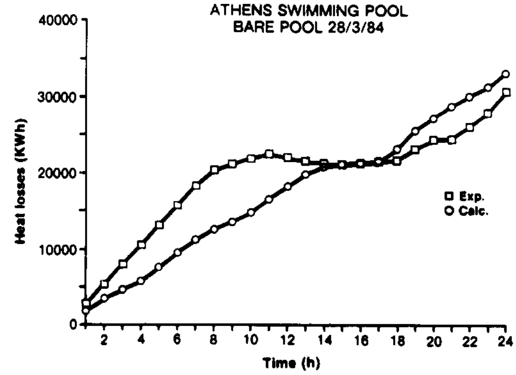

Fig. 3. Cumulative heat loss diagram.

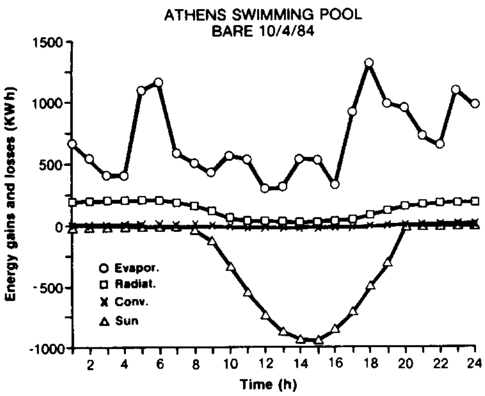

Fig. 4. Energy (hourly) gains and losses diagram.

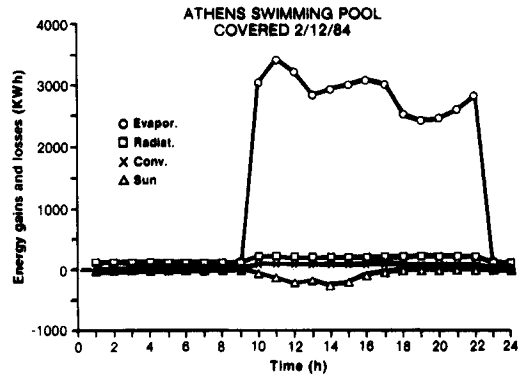

Fig. 5. Energy (hourly) gains and losses diagram.

comparison on an hourly basis, which was taken up at 1 day's period.

In Figs 4 and 5, the various energy flux components ($\dot{g}$, $\dot{q}_r$, $\dot{q}_c$, $\dot{q}_e$) are shown for 2 days, on an hourly basis, in diagrammatic form. They represent typical days, with the pool uncovered and covered (at night), respectively, and reveal that the most important loss contributor is the evaporation, which is effectively cut down by the placement of covers.

Figure 6 plots the experimental and theoretical cumulative heat loss values at the end of the day, for all days tested during the first year of operation of the Olympic swimming pool. The summer days are not presented, since then the system is rather self-balanced, at the acceptable pool water temperature, requiring either no heat input at all or a very small amount. From the plot of Fig. 6, it can be observed that the agreement is good for all days of the year tested, which is considered as very satisfactory, taking into account the broad variations that these days represent in terms of operating conditions.

CONCLUSIONS

From the theoretical analysis results and the experimental investigation presented in the preceding sections, the following conclusions are drawn:

—The agreement between the model calculations and experimental results is very promising, especially on a 24 h basis, proving the validity of the theoretical analysis (starting from first principles).

—From a comparison of the magnitudes of the individual heat losses, it is revealed that the higher loss contributor is evaporation, proving the high importance of using covers in such pools.

—Concerning the experimental investigation, the technical merits of using commercial solar collectors

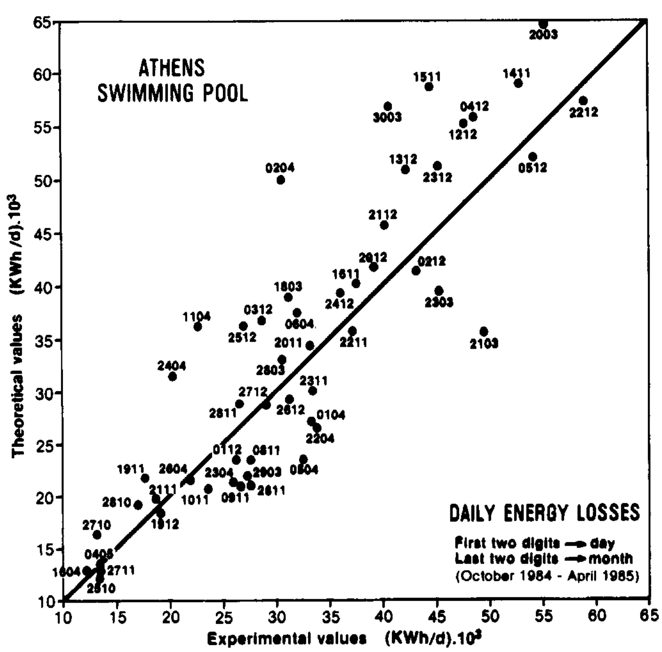

Fig. 6. Comparison of experimental and theoretical results for daily energy losses (various days).

for heating outdoor swimming pools was demonstrated, despite the amount of attention required (operational, monitoring faults).

The research, by the present group, has turned now towards suitably integrating the above equations, so that a daily model is produced. This is going to be reported in another communication.

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